Coupling of Twin Supersonic Jets of Complex Geometry

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Fundamental issues about the coupling of twin supersonic jets of complex geometry are examined. It is shown that screech tones from twin rectangular nozzles with double-beveled exit geometries can couple. Unlike coupling of twin rectangular jets of uniform geometry, the coupling here is more intricate because simultaneous multiple frequencies with a different spanwise modal structure are present. The coupling produces two frequencies, one of which is lower than the screech frequency of either jet. Although many coupling modes are kinematically permissible, the twin jets prefer two specific modes, and in some cases these two coupling modes coexist at different frequencies. Despite the geometric complexity we can effectively predict frequencies of tones from both single and twin coupled jets using the waveguide approach. It is hoped that these results and insights will assist those simulating screech for the purpose of tailoring shock-containing complex twin jets that minimize sonic fatigue failure of aircraft structures.

I. Introduction

THE resonant coupling of twin jet plumes produces very large dynamic pressures in the internozzle region that eventually causes sonic fatigue failure of aircraft structures. Several researchers have studied twin jets in an effort to solve problems with the U.S. Air Force B1-B and F15-E. This includes work at the NASA Langley Research Center, ^{1,2} by the U.S. Air Force, ^{3,4} and at McDonnell Douglas Aerospace. ^{5,6} Detailed documentation of the coupling of twin supersonic rectangular jets was provided by Raman and Taghavi in an effort to resolve numerous issues underlined by Tam and Seiner and Morris. ⁹ However, none of the cited papers addressed the coupling of twin jets of complex geometry, as this study does.

The objective was formulated on the basis of recent concerns about modern aircraft. Coupling of twin jets from modern engines that are designed for very high propulsive thrust could produce very high dynamic pressures in the internozzle region. In stealth applications, advanced materials (special type of paint or aircraft skin) used could be damaged if the dynamic pressures are very large. In addition, complex nozzles used on modern aircraft, for example, the F-22, have variable area and aspect ratios and thrust vectoring capabilities. The proper functioning of such a complex nozzle system under adverse conditions is of concern. Further, the realization that most supersonic jets will be imperfectly expanded in flight (even those from carefully designed convergent-divergent nozzles) has placed emphasis on the components of shock noise. Shock noise from supersonic jets includes both a discrete tone component called screech (see Raman¹⁰) and a broadband component that is often referred to as shock associated broadband noise (see Harper-Bourne and Fisher¹¹ and Tam¹²). Both components are undesirable due to structural and environmental concerns. Our focus here is on coupling of screech tones from twin jets of complex geometry and on providing benchmark data for validating simulations and models.

Numerous studies have focused on the flow from nozzles of nonuniform geometry. Westley and Lilley, ¹³ Powell, ¹⁴ and Lassiter and Hubbard¹⁵ investigated the use of complex nozzles for noise suppression during a period that coincided with the planning stages

of the Concorde (see Smith¹⁶). More recently elliptic,¹⁷ scarfed,¹⁸ asymmetric,¹⁹ beveled,^{20–22} and trailing-edge modifications²³ have been studied in various configurations. However, these researchers only studied single jets, whereas our focus is on the characteristics of the coupling of screech modes in twin complex nozzles. The complex nozzles considered in this study are a scaled down version of those originally developed by Rice and Raman²⁰ and by Rice²¹ for jet mixing noise reduction.

An earlier paper studied details of screech noise produced by nozzles of beveled geometries and identified several spanwise (z direction) screech modes in nozzles of nonuniform geometry.²⁴ The study indicated that the spanwise shock cell structure determined the spanwise screech mode. The various screech modes present in such complex nozzles included the spanwise symmetric mode, the spanwise oblique mode, and the spanwise antisymmetric mode (note that all screech modes were antisymmetric in the transverse y direction). Following the initial study, Tam et al.25 constructed a theoretical model that could predict the screech frequencies of single jets and mode transitions (in some cases). The waveguide approach was found to be successful in explaining the frequencies of all modes for the various nozzles. The simultaneous existence of multiple screech tones observed by Raman²⁴ (not harmonically related) was consistent with the linear assumption of the waveguide theory.²⁵ The Tam et al.25 model used later will explain the coupling of twin jets of complex geometry. A subsequent report by Raman²⁶ described unsteady aspects of shock-induced flow resonance, mode transitions, and the directivity of noise radiated from jets of complex geometry.

This paper investigates coupling of twin jets of complex geometry. Specific objectives are 1) to identify types of modes produced by the coupling of a pair of double-beveled nozzles, 2) to understand the coupling modes and compare with the Tam et al.²⁵ waveguide theory, and 3) to provide benchmark experimental data useful for validating screech simulation programs such as the ones by Cain et al.,²⁷ Cain and Bower,²⁸ Bower and Pal,²⁹ Tam and Shen,³⁰ and Tam and Thies.³¹ It is felt that the realization of these objectives would represent a step toward the prevention of sonic fatigue failure in modern aircraft.

II. Experimental Apparatus and Procedure

A. Jet Facility

The experiments were carried out at the NASA John H. Glenn Research Center at Lewis Field Jet Facility. A 76-cm-diam plenum tank was supplied with compressed air at pressures up to 875 kPa (125 psig) at 26.7°C (80°F). After passing through a filter that removed dirt and/or dust, air entered the plenum axially, where it was laterally distributed by a perforated plate and a screen. Two

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circumferential splitter rings that contained acoustic treatment (Kevlar®) removed upstream valve noise. The flow was further conditioned by two 50-mesh screens before exiting into the room through the nozzles. An automatic feedback control system maintained constant plenum pressures. The control system could restrict pressure variations during each run to within 0.2%. Such precise control was essential for this experiment because the screech tone was extremely sensitive to changes in operating conditions.

The nozzles, the probe traversing mechanism, and other reflective surfaces in the near field were covered with two layers of acoustically absorbent open-cell polyurethane foam (0.635-cm-thick uncompressed). The idea was to minimize strong reflections from the nozzles and plenum. This material is very effective in absorbing incident sound in the frequency range from 1000 to 25,000 Hz (with several layers, lower frequencies can also be absorbed).

Figure 1 shows the twin jet setup. Each nozzle included a circular to rectangular transition section and a convergent-divergent nozzle contour with exit dimensions of 3.5×0.7 cm (aspect ratio = 5). The divergence area ratio A_e/A^* was 1.128 ($M_D \simeq 1.4$). Note that the subscripte and superscript * refer to conditions at the nozzle exit and throat, respectively. M_i is used to denote the fully expanded jet Mach number. Also note that the convergence-divergence occurred only in one direction, y, with straight side walls. The bevelcuts were made at $30\,deg\,from\,the\,nozzle\,lip.\,Two\,double-bevelednozzles\,were\,located$ side-by-side with their narrow dimensions parallel and their long dimensions in the same plane. A positioning apparatus kept one of the nozzles fixed, and the other nozzle could be moved in the nozzle exit plane to achieve various internozzle spacings. Microphones mounted on the nozzles monitored the phase relationship of the screech tone at various locations. The spanwise screech modes, as well as the modes of coupling of twin jets, were determined by using microphones located at the nozzle exit plane. These are numbered as 1, 2, and 3 (see Fig. 1).

B. Measurement Techniques and Experimental Uncertainty

A spark schlieren system was used for flow visualization. The system included a Palflash light source, a microscope objective, two spherical mirrors (15.24 cm diameter, 91.44-cm focal length), and a vertical knife edge. The light source consisted of an electric arc in an inert atmosphere of argon gas and could produce a 1- μ s pulse of high-intensity light (4 J). Photographs were taken by allowing light from the knife edge to fall directly on Polaroid film.

The acoustic measurements were made in the near field using 0.64-cm- $(\frac{1}{4}$ -in.-) diam B & K microphones mounted under each nozzle and on a three-dimensional traversing mechanism for the near-field noise surveys. The uncertainty arising from the omnidirectionality of the microphones was within ± 1 dB to 10 kHz and within ± 3 dB to 20 kHz. The frequency bandwidth was 32 Hz over the frequency range from 0 to 25.6 kHz. The accuracy in the phase

measurements was ± 5 deg. The microphones were calibrated using a B & K pistonphone calibrator, with corrections for day-to-day changes in atmospheric pressure. The sound pressure levels reported are in decibel relative to 20 μ Pa. The acoustic data were recorded using a B & K analyzer.

III. Coupling of Screech Tones from Twin Jets

Coupling is defined as the phase locking of the screech instabilities of adjacent jets. A brief discussion of the differences between circular and rectangular jet coupling is in order before discussing the results. A single circular jet exhibits several modes of screech often referred to as stages A–E. In contrast, the single unbeveled rectangular jet (aspect ratio ≥ 5) has one dominant antisymmetric mode. When twin circular jets are operated simultaneously, the jets can couple in more than one way, depending on nozzle spacing and Mach number. However, the sound pressure level in the internozzle region rises dramatically when the B (helical) modes of two circular jets couple.²

In distinction, twin rectangular jets with unbeveled exits exhibit either symmetric or antisymmetric coupling. The antisymmetric coupling suppresses the sound pressure level in the internozzle region, whereas symmetric coupling augments it. For the data reported, the nozzles were located at the closest possible spacing (s/h of approximately 5.5, where h is the smaller dimension of the nozzle). This spacing produced the strongest coupling in twin rectangular jets (unbeveled) of uniform exit geometry studied by Raman and Taghavi, which were explained based on the source shift and the sudden growth of the null phase region described by Raman. However, in the present experiment with beveled nozzles, the coupling is complicated by the simultaneous presence of multiple frequencies of different mode types.

 $Tam\ and\ Thies^{31}\ \overset{\frown}{determined}\ that\ rectangular jet\ instability\ modes$ could be symmetric or antisymmetric in both the normal YZ and lateral XZ planes. Their findings were based on the wave modes in a rectangular jet that were given by the eigensolutions of the linearized governing equations. They found four linearly independent families of eigensolutions that were invariant to certain transformations. Because each of the jets has four kinematically permissible modes, many possible combinations exist when two such jets couple. Morris⁹ and Zilz and Wlezien⁶ showed that four types of coupling modes exist for twin jets of simple geometry. The probable modes based on theory of Morris⁹ and of Tam and Thies³¹ are shown in Fig. 2. The plus and minus signs represent regions that are 180 deg out of phase. Previous experiments²⁴ have shown that for rectangular jets of uniform or nonuniform geometry the screech instability is always antisymmetric in the transverse direction; therefore, only modes two and four from Tam and Thies³¹ (S1 and S2 as shown in Fig. 2) are considered for each individual jet. In the spanwise direction both antisymmetric and symmetric modes are possible. With

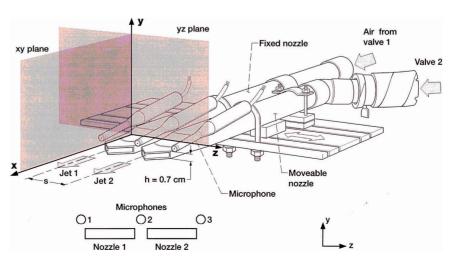


Fig. 1 Twin rectangular nozzles of nonuniform exit geometry.

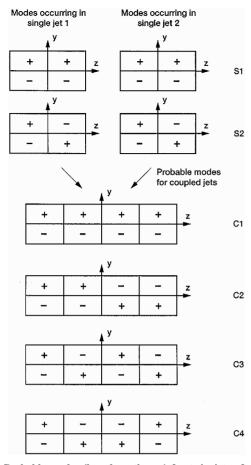


Fig. 2 Probable modes (based on theory) for twin jets of complex geometry.

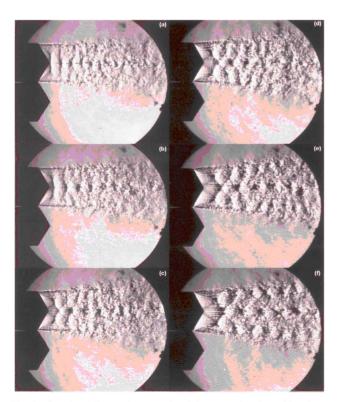


Fig. 3 Spark-schlieren photographs of the spanwise view of jets from a single double-beveled nozzle, where M_j is a) 1.2, b) 1.3, c) 1.4, d) 1.5, e) 1.6, and f) 1.68.

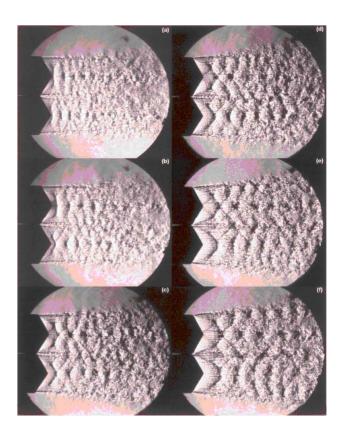


Fig. 4 Spark-schlieren photographs of the spanwise view of twin coupled jets from double-beveled nozzles, where M_j is a) 1.2, b) 1.3, c) 1.4, d) 1.5, e) 1.6, and f) 1.68.

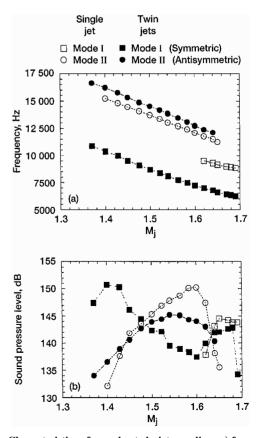


Fig. 5 Characteristics of complex twin jet coupling: a) frequency vs Mach number and b) amplitude vs Mach number.

these two possibilities (S1 and S2) for each single jet, there exist many more possibilities, for example C1–C4, for the coupled jets. In the present work we only observe coupling modes in accordance with the first two possibilities (C1 and C2). The reason why the coupled jets pick these two coupling modes even though other combinations are kinematically permissible remains an unresolved issue.

IV. Results and Discussion

Spark-schlierenphotographs (spanwise view) of single- and twinbeveled nozzles are shown in Figs. 3 and 4. The photographs document the complex shock-cell structure and the effect of coupling on the shock-cell structure of both jets over the entire Mach number range. Comparing Fig. 3 with Fig. 4 shows that the simultaneous presence of the second (bottom) jet did not alter the shock-cell structure of the top jet significantly. However, the region in between the two jets reveals that beyond $M_j = 1.4$ the jets intermingle and there

is a strong interaction and linking of the shock-cell structures of the two jets.

The frequency and amplitude of single and coupled jets are shown in Fig. 5 over a range of various levels of underexpansion. The single jet produced screech in the spanwise antisymmetric mode II between $M_j=1.35$ and 1.65. Around $M_j=1.625$, a spanwise symmetric mode I appeared. Measurements using microphones on either side of the narrow dimension of each nozzle (not shown in Fig. 1) revealed that both modes are antisymmetric in the transverse y direction. The spanwise modes were determined by measuring the phase difference between microphones 1 and 2 (for jet 1), 2 and 3 (for jet 2), and 1 and 3 (for the coupled jets). Phase differences of 0 and 180 deg corresponded to the symmetric and antisymmetric modes, respectively. An asymmetry in a shock cell located at a certain distance downstream can become tuned to satisfy the amplitude and phase criteria of Powell, 33 resulting in antisymmetric

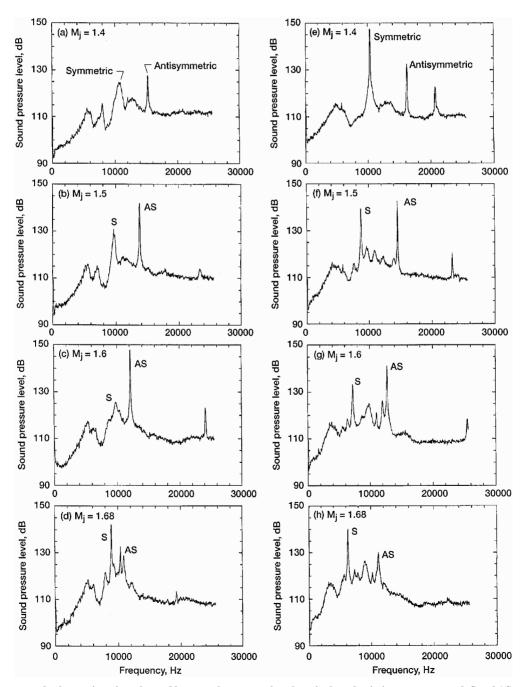


Fig. 6 Spectra measured using a microphone located between the two nozzles when single and twin jets were operated; S and AS are symmetric and antisymmetric, respectively: a) single jet, $M_j = 1.4$, b) single jet, $M_j = 1.5$, c) single jet, $M_j = 1.6$, d) single jet, $M_j = 1.68$, e) twin jets, $M_j = 1.6$, and h) twin jets, $M_j = 1.68$.

mode screech. Similarly, a spanwise symmetric shock that is located at an appropriate distance downstream, such that it satisfies Powell's criteria, will produce screech in the symmetric mode.

When both jets operated simultaneously, they could couple in either the antisymmetric II or the symmetric I mode. The modes of coupling were determined by measuring the phase difference as described earlier. When the phase difference was neither 0 nor 180 deg, the jets were not phase locked in either mode, but the weak interaction between jets could produce spanwise asymmetry in screech. For the complex double-bevelednozzles used in the present work, both coupling modes I and II were simultaneously present at two different nonharmonically related frequencies (see Fig. 5a), and strong phase locking was observed at several Mach numbers. The symmetric mode produced during coupling was much lower in frequency than that in the single jet. This observation signifies that the coupling is very strong inasmuch as it overrides the individual feedback loops of the two jets. In addition, the lower frequency symmetric mode is more likely to match a structural resonance and result in sonic fatigue failure.

Two additional points need to be made regarding the meaning of the word coupling and the role of frequency matching. First, jets are considered coupled only when their interaction results in phase-locked motions of the jet instability with possible screech amplification. A screeching jet is highly sensitive to its environment and bringing another jet close to a screeching jet would affect screech in the same way as a reflective surface might, but such interaction would hardly constitute coupling. Second, frequency itself is not a measure of coupling. For example, twin jets can be coupled at a frequency that is the same as that of each individual jet (see Raman and Taghavi⁷), or the coupled jets could screech at a frequency that is entirely different from that of either individual jet, but the coupling could still be strong for the latter case as described in the preceding paragraph.

The amplitude was measured using a microphone (microphone 2) of Fig. 1) located between the two nozzles at the nozzle exit plane. Examination of the amplitude information in Fig. 5b reveals the following information. Coupling produces a symmetric mode at low Mach numbers (1.35–1.45) that is almost 20 dB higher in amplitude than the tone from the single jet. It is clear that this amplitude increase is well above the 6 dB that one would expect through source doubling (without any coupling/interaction) and is thus the result of screech amplification arising from the coupling. As the Mach number increases from 1.45 to 1.6, coupling in the symmetric mode weakens, and the antisymmetric coupling mode dominates. However, the antisymmetric coupling generates sound pressure levels lower than that produced by the single jet. Note that an antisymmetric coupling represents a twisting mode (equivalent to an unwound helical or spiral mode in a round jet). At higher Mach numbers, symmetric mode coupling becomes more dominant than the antisymmetric mode coupling.

Spectra from a microphone midway between the nozzles (microphone 2) is shown for single and twin coupled jets (see Fig. 6). In one case (Figs. 6a–6d), only one of the two jets operated. In the second case (Figs. 6e–6h) both jets operated simultaneously. The spectra in Figs. 6a–6d indicate that for the single jet case at $M_j=1.4$ both spanwise symmetric and antisymmetric modes are equally dominant. At higher M_j the antisymmetric mode dominates (Figs. 6b and 6c). A further increase in M_j causes the symmetric mode to dominate (Fig. 6d). In contrast, the coupled jets display a dominant low-frequency symmetric coupling mode at $M_j=1.4$ (Fig. 6e). As M_j was increased both the low-frequency symmetric mode and the high-frequency antisymmetric mode were equally dominant (Figs. 6f and 6g). A further increase in Mach number ($M_j > 1.6$) caused the symmetric coupling mode to be dominant again (Fig. 6h).

At one flow condition ($M_j = 1.4$) the influence of the coupling on the linear spectral coherence between microphones located at the spanwise edges of one of the nozzles (microphones 1 and 2) is shown in Fig. 7. Because it expresses the degree of linear correlation between signals measured at two locations, $\gamma^2(f)$ is useful.

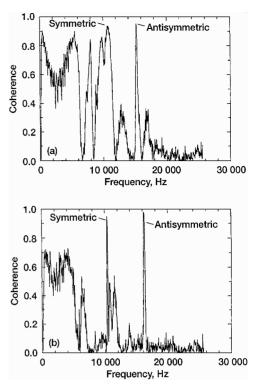


Fig. 7 Influence of coupling on the linear spectral coherence between microphones located at the spanwise edges of one of the nozzles; $M_j = 1.4$: a) single jet and b) coupled twin jets.

Whereas the single jet displays a sharply spiked coherence at the frequency of the antisymmetic mode and very broad coherence at other frequencies, the coupled jets exhibit sharply spiked coherence at both the symmetric and antisymmetric frequencies. Relatively high coherence in the low-frequency range ($f < 5000 \, \mathrm{Hz}$) is a result of coherent pressure fluctuations in a frequency band in the range of hydrodynamic instability modes growing in the jet. As shown by Raman and Taghavi⁷ for jets of uniform geometry, this low-frequency coherence exists only for closely spaced jets, and the effect disappears at jet spacings (center-to-center) beyond 9h.

Tam et al.²⁵ showed that the screech frequencies produced by single nozzles of complex geometry could be calculated using a waveguide approach. The waveguide approach also makes it possible to have several feedback loops and more than one screech tone at a given time. The major difference between screech from uniform and nonuniform nozzles is that the dominant waveguide modes that make up the quasiperiodic shock-cell structure are not necessarily the lowest order modes. Therefore, the screech frequencies can be calculated by replacing k in Tam's 12 screech frequency equation,

$$f = \frac{u_c k}{2\pi [1 + (u_c/a_{\infty})]} \tag{1}$$

with k values of higher order,

$$k_{n,m} = \left(\frac{n^2}{b_j^2} + \frac{m^2}{h_j^2}\right)^{\frac{1}{2}} \frac{\pi}{\left(M_j^2 - 1\right)^{\frac{1}{2}}}$$
(2)

where b_j and h_j are the larger and smaller fully expanded jet dimensions, respectively. Note that u_c is the convection or phase velocity of the instability waves, a_∞ is the ambient sound speed, and n and m are mode indices (integers) that represent the waveguide modes. For a large aspect ratio jet $(b_j \gg h_j)$ only the m=1 modes are relevant. The phase velocity u_c can be approximated as $0.55u_j$, where u_j is the fully expanded jet velocity.

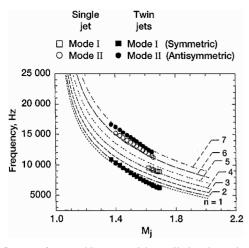


Fig. 8 Success of waveguide approach in predicting the various modes in single and coupled twin complex jets (from Tam et al. ²⁵); waveguide calculations provided by Tam and Shen (1998, private communication).

As described by Tam et al.²⁵ the screech frequency formula then becomes

$$f_n = \frac{0.55u_j}{2\pi[1 + (0.55u_j/a_\infty)]} \left(\frac{n^2}{b_j^2} + \frac{1}{h_j^2}\right)^{\frac{1}{2}} \frac{\pi}{\left(M_j^2 - 1\right)^{\frac{1}{2}}}$$
(3)

where n = 1, 2, 3, ...

Tam et al.²⁵ have shown earlier that Eq. (3) produced results in agreement with the experimental data of Raman²⁴ for single jets. In this paper, we assess the waveguide approach's utility in explaining the coupling of twin complex jets. Figure 8 shows a comparison of the Tam et al.²⁵ theory with experimental data from the present work. The calculated values for the theoretical curves in Fig. 8 were provided by Tam and Shen (private communication) and are scaled for the present set of nozzles that are smaller than those in the work of Raman.²⁴

From Fig. 8 it is seen that for a single jet from a smaller scale nozzle, the antisymmetric and symmetric modes correspond to the n=6 and 3 modes of Eq. (3). Note that, when applying the waveguide theory to the coupled twin jets, their combination is viewed as a unified jet of a larger aspect ratio. As a consequence, waveguide modes for coupled jets now correspond to the modes of coupling. When the complex twin jets couple, the antisymmetric and symmetric modes correspond to n=7 and 1, 2, respectively. Note that the symmetric mode that corresponded to the n=3 mode in single jets now follows n=2 up to $M_j=1.5$, beyond which it follows n=1. Thus, the Tam et al. 25 waveguide model is useful in explaining the anomalous symmetric mode in coupled jets as a mode jump from a higher-order waveguide mode to one that is lower.

V. Conclusions

The coupling of a pair of closely spaced rectangular jets with a double-beveled(spanwise) geometry is explored, and the following conclusions are provided.

- 1) Screech instabilities of twin jets of complex geometry could couple.
- 2) Coupling occurred at two frequencies, one of which was lower than that of either individual jet, and in a mode that was symmetric over the entire spanwise extent of both jets.
- 3) Low-frequency coupling produced a screech amplification of $20\,\mathrm{dB}$ (considerably higher than the 6 dB that one would expect due to source doubling).
- 4) Waveguide theory predicts the essential features of the frequencies of the coupling.

It is hoped that these results serve as benchmark data for those simulating the resonant coupling of twin jets of complex geometry to design nozzles that prevents onic fatigue failure of aircraft structures. Future work should focus on addressing several important effects not considered here, such as flow temperature, scalability, and flight-stream effects.

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